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Weyl gravity and galaxy clusters

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ABSTRACT

We apply a density-dependent variant of Weyl gravity, combined with Einstein-Hilbert gravity, to galaxy clusters. After fitting two global parameters, the gravity profiles calculated from the known baryonic matter in Abell 1689, Abell 1835, and Abell 2029 are compatible with the surface mass density profiles obtained from gravitational lensing and largely compatible with the acceleration profiles obtained from the hydrostatic equilibrium of the intergalactic gas, thus explaining the differences between the results of both methods. The shape of the two-dimensional κ map in 1E 0657−558 (Bullet Cluster) is reproduced qualitatively.

Keywords: dark matter, modified gravity, Weyl gravity, galaxy clusters, Abell 1689, Abell 1835, Abell 2029, 1E 0657−558, Bullet Cluster

1. INTRODUCTION

General relativity is an extremely well-tested theory which explains all phenomena related to gravity up to the distances between the stars in a galaxy (Will 2014). Measurements of gravity over galactic and extra-galactic scales, however, indicate sources of gravity which cannot be attributed to matter visible via electromagnetic waves, commonly called *baryonic matter*. This mass discrepancy leads to the conclusion that there is additional invisible matter, *dark matter* (Garrett & Duda 2011). Alternative explanations postulate modifications of the laws of gravity (Famaey & McGaugh 2012).

Both classes of explanations successfully explain some observations, for instance the rotation curves of galaxies, but have problems explaining others. In particular, modified gravity theories have problems explaining the gravitational lensing in galaxy clusters (Dutta & Islam 2018). On the other hand, all attempts to detect dark matter by other means than their gravity have produced negative results so far (Bertone et al. 2005).

An example of particular interest is the galaxy cluster 1E 0657−558, also called the *Bullet Cluster*. It consists of two colliding subclusters. Its baryonic matter components are the galaxies themselves plus intergalactic gas, whose mass is about 10 times the mass of the galaxies. The mass profile measured via gravitational lensing does not follow the profile of the baryonic mass, dominated by the gas clouds. Instead it widely follows the profile of the galaxies. This can be explained by unseen dark matter located close to the galaxies, but it is hard to explain by modified gravity, which is expected to follow the profile of the baryonic mass (Clowe et al. 2006a; Paraficz et al. 2016).

In the same galaxy cluster, however, the relative velocity of the two colliding subclusters is too high to be explained by theories involving dark matter, but it can be explained by theories of modified gravity (Angus & McGaugh 2007). In this sense, 1E 0657−558 provides direct evidence both for and against the existence of dark matter at the same time.

Another direct evidence for the existence of non-Newtonian gravity on stellar scales has been found in wide binary systems (Chae 2023).

This paper focuses on one particular theory of modified gravity, Weyl gravity (Mannheim & Kazanas 1989; Mannheim 2006; Dutta & Islam 2018), in a slightly modified form, where the gravity of a mass distribution also depends on its density, and in combination with Einstein-Hilbert gravity. We show that the Weyl gravity of the baryonic matter can explain the gravity profiles measured in the galaxy clusters Abell 1689, Abell 1835,

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Abell 2029, and 1E 0657−558. This makes Weyl gravity an interesting candidate for an explanation of the nature of dark matter.

2. WEYL GRAVITY

Weyl gravity replaces the Einstein-Hilbert action

$$S_{\mathrm{EH}} = -\frac{c^4}{16\pi G}\int (R - 2\Lambda)\sqrt{-g}\; d^4x \tag{1}$$

by the Weyl action

$$S_{\mathrm{W}} = -\alpha_g \int C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa}\sqrt{-g}\; d^4x, \tag{2}$$

where

$$\begin{aligned} C_{\lambda\mu\nu\kappa} = {} & R_{\lambda\mu\nu\kappa} \\ & - \tfrac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) \\ & + \tfrac{1}{6}R^{\alpha}{}_{\alpha}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}) \end{aligned} \tag{3}$$

is the Weyl curvature tensor and α_g is a dimensionless constant. One motivation for this arises from quantum field theory, where S_{W}, in contrast to S_{EH}, is renormalizable. Another motivation is a possible explanation for dark matter via a modified law of gravity (Mannheim & Kazanas 1989; Mannheim 2006).

The field equations emerging from S_{W} are different from the Einstein equations of gravity. Nevertheless both sets of equations allow for the Schwarzschild solution and thus the Newtonian limit, making both theories of gravity equivalent up to stellar gravity and distances.

In Weyl gravity, the classical gravitational potential of a spherically symmetric source reads

$$V_{\mathrm{W}}(r) = -\frac{c^2\beta}{r} + \tfrac{1}{2}c^2\gamma\, r + \tfrac{1}{2}c^2\gamma_0\, r - \kappa c^2 r^2, \tag{4}$$

where β and γ relate to the mass M of the source via

$$\beta = \beta_* \frac{M}{M_\odot}, \qquad \gamma = \gamma_* \frac{M}{M_\odot}, \tag{5}$$

and $M_\odot \approx 1.99\times 10^{30}$ kg is the mass of the Sun.

In Weyl gravity, the constant

$$\beta_* \approx 1.48\times 10^{3}\ \mathrm{m} \tag{6}$$

matches half the Schwarzschild radius of the Sun. Then the first term of the potential (4) reproduces the Newtonian law of gravity. The second term features a new universal constant

$$\gamma_* \approx 5.42\times 10^{-39}\ \mathrm{m}^{-1} \tag{7}$$

and describes a gravitational force which does not depend on the distance to its source.

The "global" terms involving γ_0 and κ result from masses *outside* the spherical mass distribution. On galactic scales, these constants have been determined (O'Brien & Mannheim 2012) as

$$\gamma_0 \approx 3.06\times 10^{-28}\ \mathrm{m}^{-1}, \tag{8}$$

$$\kappa \approx 9.54\times 10^{-50}\ \mathrm{m}^{-2}. \tag{9}$$

Weyl gravity modifies the law of gravity in a way which has negligible influence in the solar system, but has been shown to explain the rotation curves of galaxies (O'Brien & Mannheim 2012) and other phenomena on galactic scales (Dutta & Islam 2018). On extragalactic scales, however, the gravity generated this way by the baryonic matter is much bigger than the gravity observed indirectly via the hydrostatic equilibrium of the intergalactic gas (Horne 2006) and via gravitational lensing (Dutta & Islam 2018).

The derivation of the potential (4) from the Weyl action (2) involves integrals over the spherically symmetric mass distribution with density $\rho(r)$,

$$\beta \approx \tfrac{1}{12}\int r^4\rho(r)\, dr, \qquad \gamma \approx \tfrac{1}{2}\int r^2\rho(r)\, dr. \tag{10}$$

For a given mass distribution, the additional "Weyl mass" γ depends only on the total mass $\int r^2\rho(r)\, dr$ enclosed in a sphere, as we know it from Newtonian gravity. In contrast, the "effective Newtonian mass" β also depends on the shape of the mass distribution $\rho(r)$. This contradicts Newtonian gravity and observations (Yoon 2013). In pure (conformal) Weyl gravity this problem is solved by assuming that the gravitational mass is dominated by baryons, the protons and neutrons in the atomic nuclei. Each baryon is assumed to be a spherically symmetric source of gravity. In the limit of weak gravity we can add the contributions of all baryons to total gravity, thus restoring the independence of β of the density $\rho(r)$ (Mannheim 2016). This might, however, cause problems in the context of quantum theory when the wave functions of different baryons overlap.

3. COMBINING WEYL GRAVITY WITH EINSTEIN-HILBERT GRAVITY

In this paper, we assume the Chamseddine-Connes action (Chamseddine & Connes 1997; Connes & Marcolli 2008)

$$S_{\mathrm{CC}} = S_{\mathrm{EH}} + S_{\mathrm{W}} + S_{\mathrm{SM}}, \tag{11}$$

where the new term S_{SM} is the action of the Standard Model of elementary particles, which describes the electro-weak and strong interactions and the Higgs mechanism. (We have omitted terms which do not contribute to the equations of motion; see Connes & Marcolli (2008, § 16.1) for details.)

This action arises from the theory of noncommutative geometry, which unifies all interactions of the Standard Model with gravity by ascribing them to curvature on a noncommutative spacetime with discrete extra dimensions (Connes & Marcolli 2008; Schücker 2000, 2005). In addition to the expected terms S_{EH} and S_{SM} this action features S_{W}, thus making Weyl gravity a prediction of noncommutative geometry.

Since S_{CC} contains S_{EH}, it is not renormalizable, thus voiding one of the motivations for Weyl gravity. It is, however, possible to quantize S_{EH} and S_{CC} in the Schrödinger picture (Gerwinski 2021, 2022). This nonperturbative approach to quantization removes the need for renormalization.

The equations of motion arising from both S_{EH} and S_{W} are nonlinear. Thus it is probably out of range to find analytic solutions of the equations of motion resulting from S_{CC} in a closed form. Using reasonable assumptions we can, however, give a rough estimation of the combined gravity resulting from $S_{\mathrm{EH}}+S_{\mathrm{W}}$, Einstein-Hilbert-Weyl gravity.

Einstein-Hilbert gravity is of first order in the curvature, and Weyl gravity is of second order. In the case of very weak gravity, like between the galaxies, we can thus neglect the contribution of $C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa}$ to the action in relation to that of R and assume unmodified Einstein-Hilbert gravity. Neglecting the cosmological constant Λ, the gravity of the intergalactic gas can be described in good approximation by the Newtonian potential,

$$V_{\mathrm{gas}}(r) \approx -\frac{GM}{r}. \qquad (12)$$

When the curvature is larger, but still small enough that we can neglect the nonlinearity of Einstein-Hilbert and Weyl gravity and their coupling, we can, as a rough approximation, add up their contributions to the potential,

$$V_{\mathrm{CC}}(r) \approx -\frac{GM}{r} - \frac{c^2\beta}{r} + \tfrac{1}{2}c^2\gamma\, r. \qquad (13)$$

The parameter β no longer represents the total mass. Instead it contributes a small, density-dependent share to it, which gets overshadowed by the much bigger Newtonian mass M. Accordingly, in Einstein-Hilbert-Weyl gravity the value of the global constant β_* is much smaller than its value in "pure" Weyl gravity,

$$c^2\beta \ll GM, \qquad c^2\beta_* \ll GM_\odot. \qquad (14)$$

On the other hand the parameter γ remains unchanged and proportional to M via eq. (5). The constant γ_* is still given by eq. (7).

With these approximations we can describe the stellar gravity by the potential

$$V_*(r) \approx -\frac{GM}{r} + \tfrac{1}{2}c^2\gamma_*\frac{M}{M_\odot}\, r. \qquad (15)$$

In this potential, both the Newtonian part and the remaining "local" part of the Weyl gravity do not depend on details of the density profile $\rho(r)$. Both depend only on the total mass M enclosed in a sphere of radius $R < r$. This restores compatibility with observations without the need to assume each baryon as an individual source of gravity (Mannheim 2016).

Concerning the global terms involving the constants γ_0 and κ, we cannot assume that they have the same values on intergalactic scales as they have on galactic scales. Taking both as parameters, a (manual) fit yields

$$\gamma_0 \approx 1.68\times10^{-26}\,\mathrm{m}^{-1}, \qquad (16)$$

$$\kappa \approx 3.82\times10^{-49}\,\mathrm{m}^{-2} \qquad (17)$$

on intergalactic scales; see section 4 and 5 below.

4. RADIAL SMD PROFILES

In this section, we apply the theory of Weyl gravity, taken as an addition to Einstein-Hilbert gravity rather than a replacement, to reproduce the radial profile of the surface mass density (SMD) in Abell 1689, Abell 1835, and Abell 2029 from the baryonic matter.

Our starting point for each galaxy cluster is the set of positions and luminosities of individual galaxies as determined by the Sloan Digital Sky Survey (SDSS), data release 16. We take the r' luminosities as a relative measure for the masses of the galaxies and fit the scaling factor such that the brightest cluster galaxy (BCG) has a baryonic mass of $5\times10^{11}\,M_\odot$. For Abell 1689 this roughtly reproduces the Newtonian SMD profile of the baryonic masses of the galaxies as determined by Alamo-Martínez et al. (2013).

Since we need the three-dimensional positions of all galaxies, but we know only their right ascensions and declinations, we assign individual random distances to the galaxies using the following scheme.

- A Cartesian coordinate system is positioned with the BCG as its origin. The x and y axes correspond to right ascension and declination.
- For each cluster galaxy A except the BCG we randomly search for another cluster galaxy B of comparable mass (between 0.75 times the mass and 1.33 times the mass of galaxy A).
- We assume a cylindrical symmetry of the galaxy cluster around the x axis (which also holds approximately for 1E 0657−558) and assign the y coordinate of galaxy B to the z coordinate of galaxy A.

- We take the size of the BCG into account by dividing it into 200 point-like galaxies whose distribution roughly models an elliptic galaxy.

Using a pseudo-random number generator with known seed value, we obtain reproducible pseudo-random 3d distributions of the galaxies. All results of this work have been checked to be independent, up to negligible fluctuations, of the random seed value. In contrast, setting all z coordinates to zero does not yield plausible results.

Next we apply eq. (15) to all galaxies within a radius of 400 kpc of the BCG, taken as point sources, and calculate the gravitational acceleration caused by the baryonic matter, as felt by a test particle. We use the divergence of the resulting vector field to turn this acceleration profile into an SMD profile. To make it comparable to SMD profiles obtained using gravitational lensing, we have to take into account that the gravitational lensing of Weyl gravity is smaller than the one caused by Einstein-Hilbert gravity by a factor of 4 (O'Brien et al. 2017).

This factor is only known for the local part of the Weyl gravity. For the global part we assume the same factor of 4 for the κ term. (This compensates, probably by chance, the fitted intergalactical value of κ which is four times its galactical value.) Concerning the γ_0 term, assuming a factor of 4 again does not lead to plausible results, so we use a factor of 1. (Other values for this factor lead to different fits for γ_0, but to very similar SMD profiles.)

Although the total mass of the intergalactic gas is much bigger than the total mass of the galaxies, its Einstein-Hilbert gravity is, for all galaxy clusters considered, just a small fraction of the Weyl gravity of the galaxies.

Figure 1, 2, and 3 show the radial SMD profiles calculated this way for Abell 1689, Abell 1835, and Abell 2029, compared to the SMD profiles obtained from astronomical observations via gravitational lensing by Nieuwenhuizen et al. (2021), Alamo-Martínez et al. (2013), Umetsu et al. (2005), Broadhurst et al. (2005), Broadhurst et al. (2008) and Ménard et al. (2002).

For Abell 1689 and Abell 1835, the resemblance is obvious. For Abell 2029, the similarity fits in with the uncertainty of the observational data. A possible reason for the deviations at large radii is that we are only considering galaxies within a radius of 400 kpc from the BCG. Increasing this radius to include more cluster galaxies also means to include more non-cluster galaxies into the calculations, which leads to unrealistically high gravity.

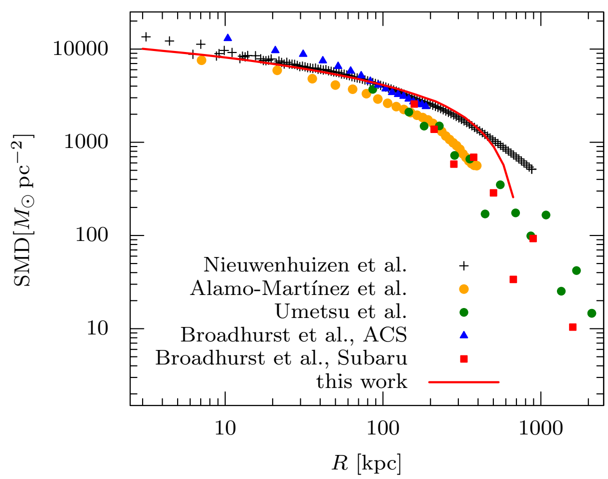

Figure 1. Abell 1689: radial SMD profiles

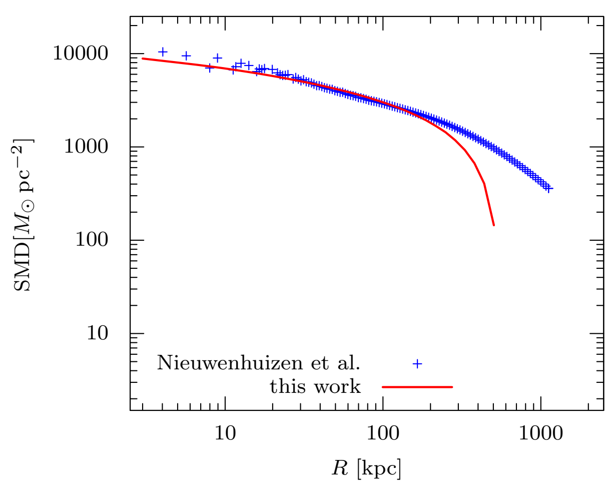

Figure 2. Abell 1835: radial SMD profiles

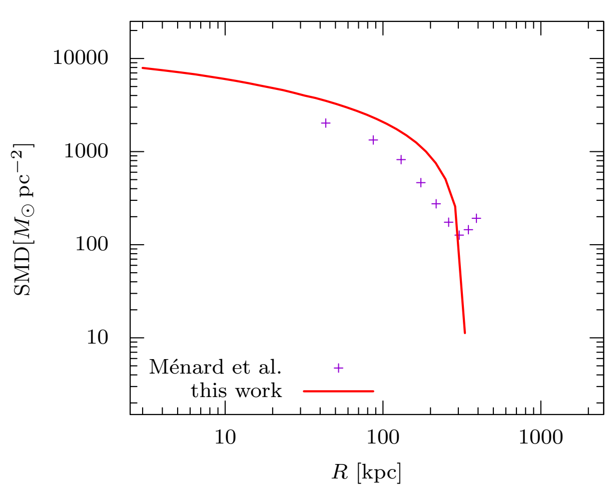

Figure 3. Abell 2029: radial SMD profiles

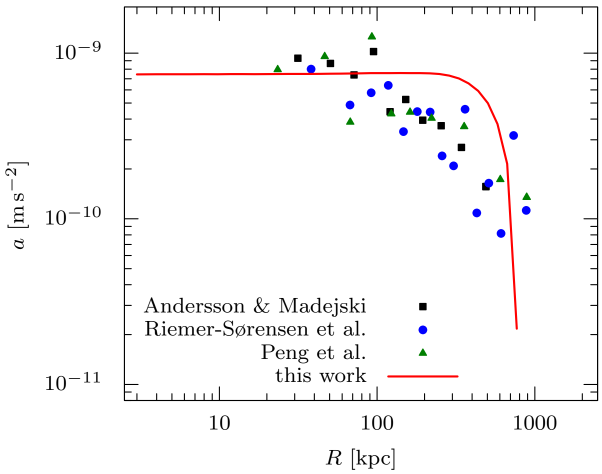

Figure 4. Abell 1689: radial acceleration profiles

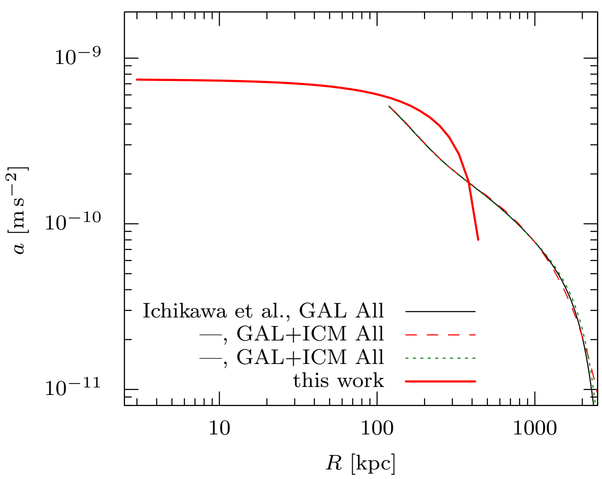

Figure 5. Abell 1835: radial acceleration profiles

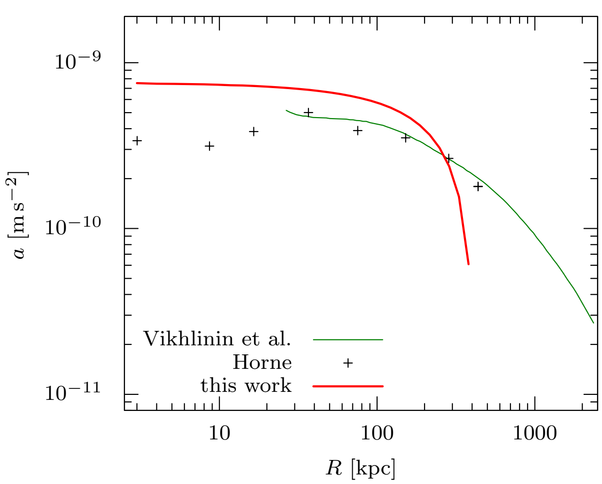

Figure 6. Abell 2029: radial acceleration profiles

5. RADIAL ACCELERATION PROFILES

Another observational approach to determine the gravity in galaxy clusters is to assume hydrostatic equilibrium of the intergalactical gas and to deduce the gravitational acceleration from X-ray observations.

Since the effects of Weyl gravity on gravitational lensing and on test particles such as gas atoms are different, we cannot directly compare our results for SMD profiles with acceleration profiles obtained via hydrostatic equilibrium by translating between the acceleration and the SMD. Instead, we directly extract the radial component from our calculated gravitational vector field.

Figure 4, 5, and 6 show the radial acceleration profiles calculated this way for Abell 1689, Abell 1835, and Abell 2029, compared to the acceleration profiles obtained from astronomical observations via the hydrostatic equilibrium of the intergalactical gas by Andersson & Madejski (2004), Riemer-Sørensen et al. (2009), Peng et al. (2009), Ichikawa et al. (2013), Vikhlinin et al. (2006), and Horne (2006).

The similarities are less striking than when comparing SMD profiles, but the calculated profiles are in the correct range to match the observed ones. This is a possible solution to the open problem why the SMD profiles observed via gravitational lensing do not match the acceleration profiles observed via hydrostatic equilibrium (Andersson & Madejski 2004; Peng et al. 2009): The effects of Weyl gravity on gravitational lensing and on gas atoms are different.

6. 1E 0657−558: 2D SMD PROFILE

The galaxy cluster 1E 0657−558 (Bullet Cluster) is of particular interest because its SMD profile measured via gravitational lensing (κ map) does not follow the SMD profile of the baryonic mass, dominated by the intergalactic gas clouds. Instead, it appears to follow the SMD profile of the galaxies alone, without the gas clouds.

In the context of combined Einstein-Hilbert-Weyl gravity this is not a surprise. On extragalactic scales, Weyl gravity dominates over Einstein-Hilbert gravity. The gas clouds, albeit heavier than the galaxies, are much less dense than the stars in the galaxies and thus do not significantly contribute to Weyl gravity.

Our calculations for 1E 0657−558 largely follow those for Abell 1689, Abell 1835, and Abell 2029. Notable differences are:

- There are no SDSS data for 1E 0657−558. Instead, the source of our data for the baryonic mass – and of the κ map shown in fig. 7 – is Clowe et al. (2006a), data release 1 (Clowe et al. 2006b).

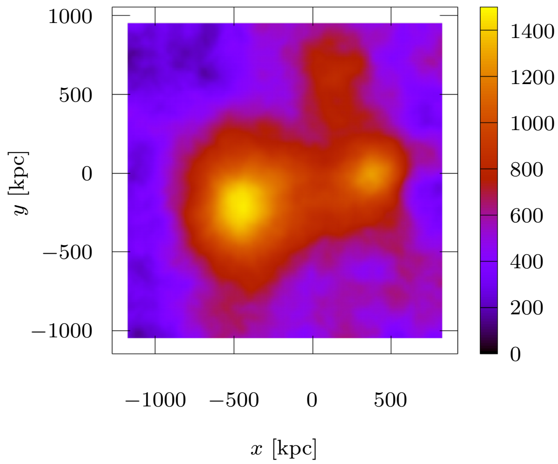

Figure 7. 1E 0657−558: two-dimensional κ map according to Clowe et al. (2006a), data release 1 (Clowe et al. 2006b)

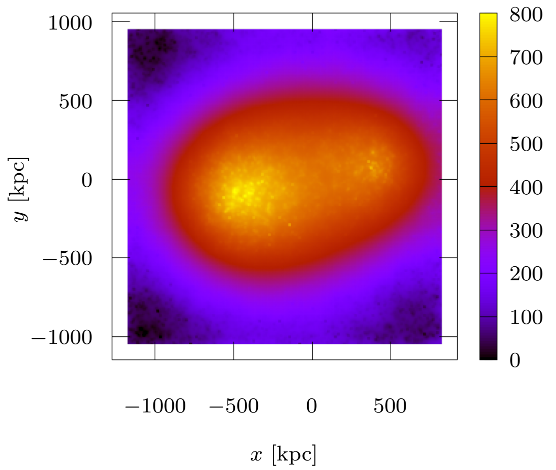

Figure 8. 1E 0657−558: two-dimensional SMD profile as calculated in this work

- Instead of considering all galaxies within a radius of 400 kpc of the BCG, we separate the main cluster and the subcluster by considering all galaxies within a radius of 160 kpc around the BCG of the main cluster and within a radius of 120 kpc around the BCG of the subcluster.

- Since a spherically symmetric mass distribution is a poor approximation for 1E 0657−558, the global contributions to Weyl gravity are in fact unknown and have been omitted in our calculations for 1E 0657−558.

- We neglect the size of the BCG and take it as a single point-like galaxy instead of dividing it into point-like subgalaxies.

Figure 8 shows a two-dimensional SMD profile, calculated for 1E 0657−558 by probing for the SMD at 10000 pseudo-randomly selected points in the vicinity of the galaxy cluster and interpolating between them by taking a weighted average of the input data at each grid point, weighted by the inverse of the distance to the data point. The result bears indeed some qualitative resemblance with the κ map observed by Clowe et al. (2006a) via gravitational lensing as shown in fig. 7.

7. CONCLUSIONS AND OUTLOOK

So far, theories of a modified law of gravity have explained many aspects of the phenomenon known as *dark matter*, but they could not explain the gravity profiles in galaxy clusters. Density-dependent Weyl gravity, combined with Einstein-Hilbert gravity, closes this gap.

The different effect of Weyl gravity on test particles and on gravitational lensing is a possible explanation for the differences between gravity profiles measured via the hydrostatic equilibrium of the intergalactic gas and via gravitational lensing in Abell 1689 (Andersson & Madejski 2004; Peng et al. 2009).

Since other aspects of dark matter such as rotation curves of galaxies are dominated by the stars, not the gas, it should be possible to transfer the success of previous calculations involving Weyl gravity (O'Brien & Mannheim 2012; Dutta & Islam 2018) to combined Einstein-Hilbert-Weyl gravity without major modifications. This makes Einstein-Hilbert-Weyl an interesting candidate for explaining all aspects of dark matter.

There is, of course, room for further improvement.

- So far, the graviational law of Einstein-Hilbert-Weyl gravity, eq. (13), is just a rough estimation. A natural next step would be to derive the precise form of this potential from the Chamseddine-Connes action, eq. (11).

- The mass profiles of the galaxies in the clusters, which are the starting points for all calculations of gravity profiles in this paper, are also rough estimations and should be refined.

- The factors between the impact of Weyl gravity on the gravitational acceleration of a test particle and on gravitational lensing, currently just guessed based on results by O'Brien et al. (2017), should be derived from the Chamseddine-Connes action, eq. (11), as well.

- This theory allows for a more direct comparison of its results with observations via gravitational lensing. Instead of calculating one- or two-dimensional profiles, we could calculate the Einstein-Hilbert-Weyl gravity dedicatedly at those positions where the gravitational lensing has been measured.

If this theory proves correct, one possible application might be the calculation of the relative distances of the individual galaxies in a cluster from the two-dimensional gravity profile of dark matter, measured via gravitational lensing (Ghosh et al. 2023).

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